Risk Parity Notes

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July 2026

1 Intro

Assume we have a portfolio with n assets and the weights are w, an n-by-1 vector, i.e., w_i denotes the weight of asset i. Assume that the variance-covariance (VCV) matrix is S with the diagonal elements $S_{i,i}$ denoting the asset variances and the off-diagonal elements $S_{i,j}, i \neq j$ denoting the covariances.

Also assume that the weights sum to one, i.e., $\sum_{i=1}^{n} w_i = 1$. The portfolio variance is then

$$V(w) = w' \cdot S \cdot w = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j S_{i,j} = \sum_{i=1}^{n} \left(w_i^2 S_{i,i} + \sum_{j \neq i} w_i w_j S_{i,j} \right) \tag{1}$$

Of course, the portfolio risk, or standard deviation is simply the square root of the variance:

$$R(w) = \sqrt{V(w)} \tag{2}$$

2 Risk Attribution

2.1 Asset i's Contribution to the Portfolio Variance

Let's call A_i the (absolute) contribution of asset i to the total variance:

$$A_i = \sum_{j=1}^{n} w_i w_j S_{i,j} = w_i^2 S_{i,i} + \sum_{j \neq i} w_i w_j S_{i,j} \tag{3}$$

Notice that all the covariance terms in the total variance show up with a multiplier of 2 because $w_i w_j S_{i,j} = w_j w_i S_{j,i}$ due to the symmetry in the variance-variance matrix. So, in effect, the Absolute Contribution assumes that each asset i contributes its own variance $w_i^2 S_{i,i}$ plus one half of the covariance contributions it shares with all the other assets.

The **share** of the variance coming from asset i is then $\frac{A_i}{V(w)}$.

A simple example with two assets: Stocks and bonds have standard deviations σ_s, σ_b, respectively. Their correlation is ρ, hence the covariance is $\rho\sigma_s\sigma_b$. The variance is:

$$V(w_s, w_b) = w_s^2\sigma_s^2 + w_b^2\sigma_b^2 + 2w_s w_b \rho\sigma_s\sigma_b \tag{4}$$

The part attributable to stocks is $w_s^2\sigma_s^2 + w_s w_b \rho\sigma_s\sigma_b$, while $w_b^2\sigma_b^2 + w_s w_b \rho\sigma_s\sigma_b$ is due to bonds. We can also calculate the share attributable to stocks and bonds:

$$C_s = \frac{w_s^2\sigma_s^2 + w_s w_b \rho\sigma_s\sigma_b}{w_s^2\sigma_s^2 + w_b^2\sigma_b^2 + 2w_s w_b \rho\sigma_s\sigma_b}, C_b = 1 - C_s \tag{5}$$

A numerical example: Assume $\sigma_s = 0.16$, $\sigma_b = 0.06$, and $\rho = 0.10$. In a 60% stocks, 40% bonds portfolio portfolio, the equity contribution to the portfolio variance is:

$$C_s = \frac{0.16^2 \cdot 0.6^2 + 0.6 \cdot 0.4 \cdot 0.1 \cdot 0.16 \cdot 0.06}{0.16^2 \cdot 0.6^2 + 0.06^2 \cdot 0.4^2 + 2 \cdot 0.6 \cdot 0.4 \cdot 0.1 \cdot 0.16 \cdot 0.06} = 0.9213 \tag{6}$$

Thus, a 60% equity porfolio still receives 92.13% of its variance from equities.

Implementation in Matlab, Python, etc.:

1. Create a matrix W with $W_{i,j} = w_i \cdot w_j$. In Matlab: **W=weights*weights'**

2. Multiply with the VCV element-by-element with W and sum the columns: In Matlab: **WS=sum(S.*W)**

3. Normalize the sum of the contributions to one. In Matlab: **C=WS/sum(WS)**

The implementation in Python is similar, but notice that the numpy toolkit commands are **np.dot** for the standard matrix algebra multipliction and **np.multiply** for the element-by-element multiplication.

2.2 Asset i's Marginal Contribution to the Portfolio Risk

Portfolio risk $R(w) = \sqrt{V(w)}$ is homogeneous of degree one with respect to the weights, i.e., $R(\lambda w) = \lambda R(w)$, for any scalar λ. By Euler's Theorem, we know that $R(w) = \sum_i w_i \frac{\partial}{\partial w_i} R(w)$, i.e., if we multiply each asset i's marginal impact on Risk with asset i's weight, then we can interpret this measure as asset i's contribution to the overall portfolio risk.

Notice that $\frac{\partial}{\partial w_i} R(w) = \frac{1}{2R(w)} \left(2 w_i S_{i,i} + \sum_{j \neq i} 2 w_j S_{i,j}\right) = \frac{1}{R(w)} \left(w_i S_{i,i} + \sum_{j \neq i} w_j S_{i,j}\right)$. If we multiply this term by w_i, then this becomes $\frac{A_i}{R(w)}$. If we normalize all contributions to one again, we'd find that each asset i contributes a share of $\frac{A_i}{V(w)}$ to the total risk. The same share as with the Portfolio Variance method above.

3 Risk Parity

Risk parity weights are such that all assets contribute the same share to the total variance or risk, i.e., we pick weights w_{rp} such that

$$\frac{A_i}{V(w)} = \frac{1}{n}$$

These are n equations in n unknowns. We can solve this system of equations with a root solver, e.g., **fsolve** in Matlab. Alternatively, one could minimize the squared deviations from the target contributions using **fmincon**. Similar tools are available in Python, for example, through the SciPy toolbox.

4 Any Risk Target

There is nothing special about Risk Parity, where all assets get the same relative risk budget. Sometimes we want to target weights very different from Risk Parity. For example, if we have five different equity indexes and one bond index, we may assign 10% risk budget to each equity index and 50% to the bond index. Strict Risk Parity would create a portfolio with 83.3% (=5/6) risk coming from quities.